

# Quiz 2, MATH 240, Fall 2023

Write your name clearly.

Name:

Section Number:

UID:

(1) Consider the following system of equations:

$$\begin{aligned}x_1 + 2x_2 - x_3 &= 3 \\ -2x_1 - 3x_2 + 3x_3 &= 3 \\ 2x_1 + 5x_2 - x_3 &= 15\end{aligned}$$

(a) [3] Express the system as the matrix equation  $A\mathbf{x} = \mathbf{b}$ .

(b) [12] Find the solution set to the system.

(c) [5] Write  $\begin{pmatrix} 3 \\ 3 \\ 15 \end{pmatrix}$  as a linear combination of  $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ ,  $\begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$ , and  $\begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$ . You should have numbers for your weights, no variables.

(a) 
$$A = \begin{pmatrix} 1 & 2 & -1 \\ -2 & -3 & 3 \\ 2 & 5 & -1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 3 \\ 3 \\ 15 \end{pmatrix}.$$

(b) 
$$\left[ \begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ -2 & -3 & 3 & 3 \\ 2 & 5 & -1 & 15 \end{array} \right] \xrightarrow{\text{RREF}} \left[ \begin{array}{ccc|c} 1 & 0 & -3 & -15 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

The solutions are given by

$$x_1 = 3x_3 - 15$$

$$x_2 = -x_3 + 9$$

$$x_3 \text{ free}$$

(c) Take  $x_3 = 0$ . Then  $x_1 = -15$ ,  $x_2 = 9$ .

$$\text{So } \begin{pmatrix} 3 \\ 3 \\ 15 \end{pmatrix} = -15 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} + 9 \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}.$$